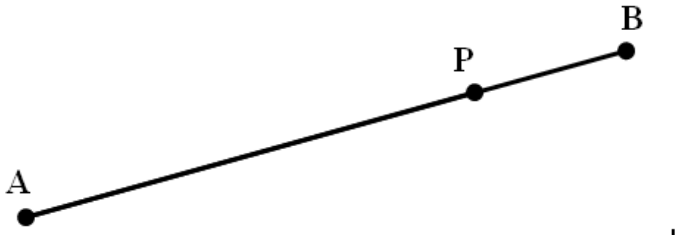
Lesson 3: Partitioning a Line Segment

Standard: G.GPE.4: Use coordinates to prove simple geometric theorems algebraically. **Standard: G.GPE.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Essential Question: How can a line be partitioned? How do you find the point of a directed line segment that partitions the segment in a given ratio?

Point P divides $\overline{AB}$ in the ratio 3 to 1.

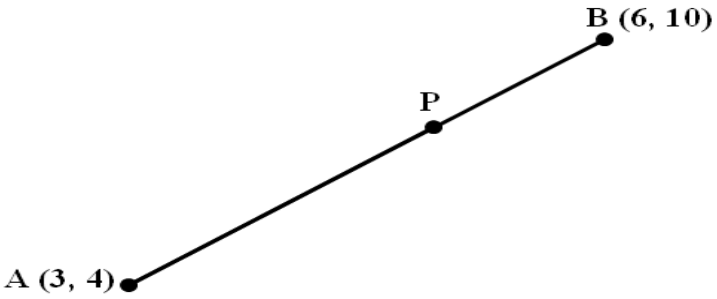
1. What does this mean? Prove it!



2. Do you expect point P to be closer to A or closer to B? Why?

3. How does the slope of $\overline{AP}$ compare with slope of $\overline{AB}$? Why?

Find the coordinate of point *P* that lies along the directed line segment from *A* (3, 4) to *B* (6, 10) and partitions the segment in the ratio of 3 to 2.



A ***directed line segment*** means the line segments has a direction associated with it, usually specified by moving from one endpoint to the other.
Tells the direction in which from which point to start and end. In this case, from Point A to Point B, therefore point A must be labeled A (x_1, y_1) and B (x_2, y_2)

What does that tell you about the distance AP and PB in relation to AB?

1. Label your points (x_1, y_1) and B (x_2, y_2)
   Note: since it is a directed segment, **order does matter**.
2. 2. Convert the ratio into a percent (keep as a fraction) ***a:b***
   $Percent\ ratio\ (\%) = \frac{a}{a+b}$
3. Find the rise and run for the segment (**order does matter**)
   $rise: y_2 - y_1 \quad run: x_2 - x_1$
4. To find the partitioning point:
   $x - coordinate: \boldsymbol{x_1} + \boldsymbol{run}\ (\%\ \boldsymbol{in\ fraction\ form})$
   $y - coordinate: \boldsymbol{y_1} + \boldsymbol{rise}\ (\%\ \boldsymbol{in\ fraction\ form})$

How can you use the distance formula to check that *P* partitions $\overline{AB}$ in the ratio of 3 to 2?

Coordinates of point which partitions a directed line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

$$(x, y) = \frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}$$

OR

$$(x, y) = \left(x_1 + \frac{a}{a + b}(x_2 - x_1), y_1 + \frac{a}{a + b}(y_2 - y_1)\right)$$

Example 1:

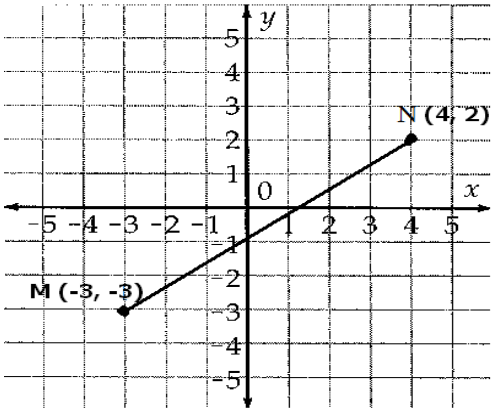
Find the coordinates of the point P that lies along the directed segment from A (1, 1) to B (7, 3) and partitions the segment in the ratio of 1 to 4

Example 2:

Find the coordinate of the point P that lies along the directed segment from C (-3, -2) to D(6,1) and partitions the segment in the ratio 2 to 1.

Example 3:

Find the coordinates of point P that lies along the directed line segment from M to N and partitions the segment in the ratio of 3 to 2



Steps:

I. Label you points (x_1, y_1) and (x_2, y_2) **Note: since it is a directed segment, **order does matter.**

II. Convert the ratio to a fraction. Ratio: *a: b* Percent ratio: $\frac{a}{a+b}$

III. Find the Δ x and the Δ y: Δ x (run) $\Rightarrow x_2 - x_1$

Δ y (rise) $\Rightarrow y_2 - y_1$

IV. To find the partitioning point:

x: x_1 + run (percent ratio)

y: y_1 + rise (percent ratio)

V. Write final values as an (x, y) ordered pair.

Coordinates of point which partitions a directed line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

$$(x, y) = \frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}$$

OR

$$(x, y) = \left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1)\right)$$

HOMEWORK

1. Find the coordinates of point P that is $\frac{3}{4}$ of the way along the directed line segment from C (6, -5) to D (-3, 4).

2. Find the coordinates of point Q that is $\frac{2}{3}$ of the way along the directed segment from R (-7, -2) to S (2, 4).

3. Find the coordinates of the point R that lies along the directed segment from J (10, -5) to K (-2, -3) and partitions the segment in the ratio of 2 to 7.

4. Find the coordinates of the point P that lies along the directed segment from M (-5, -2) to N (-5, 8) and partitions the segment in the ratio of 4 to 6.

Practice Quiz 2 Unit 5- Partitioning a Line Segment

Standard: G.GPE.4: Use coordinates to prove simple geometric theorems algebraically. **G.GPE.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

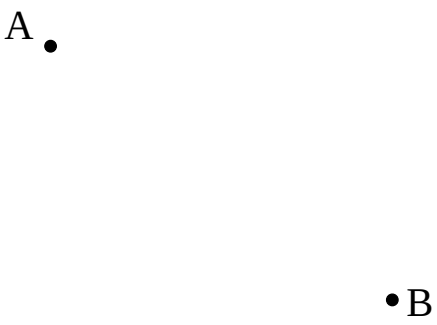
Coordinates of point which partitions a directed line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

$$(x, y) = \frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}$$

OR

$$(x, y) = \left(x_1 + \frac{a}{a + b}(x_2 - x_1), y_1 + \frac{a}{a + b}(y_2 - y_1)\right)$$

1. Find the coordinates of the points that divides $\overline{AB}$ into three equal parts.



2. What point along the directed segment from A to B partitions the segment in the ratio 1 to 2?

3. Find the coordinates of point P, that lies $\frac{2}{3}$ of the way on the directed line segment $\overline{AB}$, where A(-2, 5), B(4, 9)

4. Find the coordinates of point P that lies on the line segment $\overline{MQ}$, M(-9, -5), Q(3, 5), and partitions the segment at a ratio of 2 to 5

5. Find the coordinates of point P that lies along the directed segment from T(2,-6) to Q (-8, -4) and partitions the segment in the ratio of 3:4

6. Find the coordinates of point P that is two thirds of the way from point A(9, 6) to point B(-9, 6).

7. Given the points A(-3, -4) and B(5, 0), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 2:3.